QUIZ #3 M427L SUMMER 2005: SPENCER STIRLING (PROF. FRIEDMAN)

Instructions: this is a multiple choice exam. Please circle the correct answer. At the end of the exam please hand in your work along with this sheet (answers marked without work shown will not be considered). Please organize your work so that it is legible!

Problem 1

Consider the region $T(A)$ bounded by the u-axis, the curve $v = e^u$, and the lines $u = 0$ and $u = 1$. Assume that this region is the image of a region A which lives in the first quadrant of the xy-plane, and assume that the map is given by $T(x, y) = (y^2, x^3)$. If I ask you to calculate the integral $\int_{T(A)} e^{u^2+v^2} dudv$ by instead performing an integral in the xy-plane, which one of these integrals gives the same result?

a) $\int_A 6x^2 y e^{y^4+x^6} dxdy$
b) $\int_A \frac{1}{x^2 y} e^{y^4+x^6} dxdy$
c) $\int_A (y^4 + x^6) e^{6x^2 y} dxdy$
d) NONE OF THESE

Problem 2

What is the value of the integral $\int_{\overrightarrow{c}} yzdc$ where $\overrightarrow{c}(t) = (t, 3t, 2t)$ is a path in (x, y, z)-space going from $t = 1$ to $t = 3$?

a) $52\sqrt{14}$
b) $120\sqrt{14}$
c) $6\sqrt{7}$
d) NONE OF THESE

Problem 3

What is the value of the integral $\int_{\overrightarrow{c}} \overrightarrow{F} \cdot \overrightarrow{dc}$ where $\overrightarrow{F} = (x, y, z)$ is a vector field and $\overrightarrow{c}(t)$ is the path as given in Problem 2?

a) $\frac{364}{3}$
b) 0
c) 56
d) NONE OF THESE

Problem 4

Which of the following is true? (here ∂ means "boundary of")?

a) $\int_{\partial A} (\nabla \times \overrightarrow{F}) \cdot \overrightarrow{ds} = \int_A \overrightarrow{F} \cdot \overrightarrow{n} dA$
b) $\int_{\partial V} \nabla \cdot \overrightarrow{F} dA = \int_V \overrightarrow{F} dV$
c) $\int_V \nabla \cdot \overrightarrow{F} dV = \int_{\partial V} \overrightarrow{F} \cdot \overrightarrow{n} dA$
d) NONE OF THESE

Problem 5

What is the value of the line integral $\int_C (2x^3 - y^3)dx + (x^3 + y^3)dy$ where C is the unit circle?

a) $\frac{3\pi}{2}$
b) 27π
c) $2\sqrt{2}\pi$
d) NONE OF THESE

Problem 6

For the vector field $\overrightarrow{F} = (1, 2, z^3)$ what is the value of the integral $\int_A (\nabla \times \overrightarrow{F}) \cdot \overrightarrow{n} dA$ where A is defined by the half-sphere $x^2 + y^2 + z^2 = 1$ and $z \geq 0$?

a) 0
b) $\frac{3}{2}$
c) $\frac{\sqrt{3}}{4}$
d) NONE OF THESE