HOMEWORK #8 (M427K FALL 2004)

Introduction

This homework comes from the textbook (Boyce). You are supposed to solve problems 6, 7, and 8 on page 60-61

1. Page 60 #6

Suppose that a tank containing a certain liquid has an outlet near the bottom. Let $h(t)$ be the height of the liquid surface above the outlet at time t. Torricelli's principle states that the outflow velocity v at the outlet is equal to the velocity of a particle falling freely (with no drag) from the height h.

a) Show that $v = \sqrt{2gh}$, where g is the acceleration due to gravity

b) By equating the rate of outflow to the rate of change of liquid in the tank, show that $h(t)$ satisfies the equation

$$A(h)\frac{dh}{dt} = -\alpha a\sqrt{2gh},$$

where $A(h)$ is the area of the cross section of the tank at height h and a is the area of the outlet. The constant $\alpha$is a contraction coefficient that accounts for the observed fact that the cross section of the (smooth) outflow stream is smaller than a. The value of α for water is about 0.6.

c) Consider a water tank in the form of a right circular cylinder that is 3 m high above the outlet. The radius of the tank is 1 m and the radius of the circular outlet is 0.1 m. If the tank is initially full of water, determine how long it takes to drain the tank down to the level of the outlet.

2. Page 60 # 7

Suppose that a sum S_0 is invested at an annual rate of return r compounded continuously.

a) Find the time T required for the original sum to double in value as a function of r.

b) Determine T if r =7%.

c) Find the return rate that must be achieved if the initial investment is to double in 8 years.

3. Page 60 #8

A young person with no initial capital invests k dollars per year at an annual rate of return r. Assume that investments are made continuously and that the return is compounded continuously.

a) Determine the sum $S(t)$ accumulated at any time t.

b) If r =7.5%, determine k so that \$1 million will be available for retirement in 40 years.

c) If $k = \$2000/\text{year}$, determine the return rate r that must be obtained to have \$1 million available in 40 years.