HOMEWORK #14 (M427K FALL 2004)

Introduction

This homework is meant for you to figure out what linear independence/dependence is.

1. Is This Linearly Independent or Dependent?

$$\{x, 4x^2\}$$

2. Is This Linearly Independent or Dependent?

$$\{1, x, x^2, x^3, \ldots, x^5\}$$

(Hint: Linearly independent. You can try the Wronskian method to show it, although that means that you have to take the determinant of a 5x5 matrix. You can also use an argument about the numbers of times any polynomial can cross the x-axis (this falls into the "trickery" category)

3. Is This Linearly Independent or Dependent?

$$\{1, x, x^3, x^3, \ldots, x^{1000}\}$$

3.1. **Solution.** These are dependent. To prove it, I can just write the equation

$$C_0(1) + C_1 x + C_2 x^3 + C_3 x^3 + \cdots C_{1000} x^{1000} = 0.$$

Now If I let $C_1 = 1$ and $C_2 = -1$ and all of the other $C_i = 0$ then this is clearly a solution to this equation without ALL of the C's zero. So it's linearly DEPENDENT. End of story.

Question: how would I deal with this problem if the first x^3 had really been an x^2? Then the set would've been INDEPENDENT, but I wouldn't have wanted to use the Wronskian to prove it. Think about why not?

4. Is This Linearly Independent or Dependent?

$$\{e^{2x}, e^{3x}\}$$

5. Is This Linearly Independent or Dependent?

$$\{x, e^x, xe^x\}$$