HOMEWORK #12 (M427K FALL 2004)

INTRODUCTION

This homework is meant for you to explore the Heaviside formulas $6'$ and $7'$, which are given by the equations

$$\frac{1}{D^2 + a^2} \sin(ax) = -\frac{x}{2a} \cos(ax)$$

$$\frac{1}{D^2 + a^2} \cos(ax) = \frac{x}{2a} \sin(ax)$$

1. VERIFY THAT EQUATION $7'$ IS RIGHT

(Hint: try applying $(D^2 + a^2)$ to both sides and see if you get a true equation)

2. VERIFY EQUATION $6'$ USING EQUATION $5'$

(Hint: First use Euler's equation $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ to rewrite the sin in the following way:

$$\frac{1}{D^2 + a^2} \sin(ax) = \frac{1}{D^2 + a^2} \left(\frac{e^{aix} - e^{-aix}}{2i} \right).$$

Then break up the denominator of the operator as

$$\frac{1}{(D + ai)(D - ai)} \left(\frac{e^{aix} - e^{-aix}}{2i} \right).$$

Now apply Heaviside equation $5'$).

3. VERIFY EQUATION $7'$ USING EQUATION $5'$

(Hint: almost exactly the same as the previous exercise, but this time we have a cos).