# M1090 midterm 1 (Spencer Stirling) - February 17, 2010

Directions: Use both the front and back of the paper for your solutions. You may attach more sheets if necessary. SHOW ALL WORK and CLEARLY mark your solutions.

1) (2 points) Solve the following equation. Check the domain to ensure that the solution is valid.

$$\frac{2x-1}{x+5}+3=\frac{7}{x+5}$$

$x+5=0$ NOT in domain, i.e. when $x=-5$ this is undefined

$\rightarrow 2x-1+3(x+5)=7 \rightarrow 2x-1+3x+15=7 \rightarrow$

$\hookrightarrow 5x=-7 \rightarrow x=-\frac{7}{5}$ ← this is not $-5$, so it is the solution

2) (3 points) INEQUALITIES: Your new job allows you to choose between two ways of being paid. When is the first way better than the second?

| | Monthly Base Salary | Sales Commission |
|---|---|---|
| (1) | \$700 | 4% of all sales |
| (2) | \$1000 | 3% of all sales |

$\text{Income}_1 = 0.04x + 700$ $x$ = sales (in dollars)

$\text{Income}_2 = 0.03x + 1000$

$\text{Income}_1 > \text{Income}_2$ (note: I will accept $\geq$ since "better" isn't precise)

$0.04x + 700 > 0.03x + 1000$

$\rightarrow 0.01x > 300 \rightarrow x > \frac{300}{0.01} \rightarrow x > \$30{,}000$ in sales

this is when the first deal is better than second

3) (3 points) Write the equation for a line that passes through the given point and is *perpendicular* to the given line

point is $(2,3)$ and line is $3x + 4y = 7$

given line $3x+4y=7 \rightarrow y = -\frac{3}{4}x + \frac{7}{4}$ slope $= -\frac{3}{4}$, perp slope $= \frac{4}{3}$

$y - 3 = \frac{4}{3}(x-2)$ simplify: $y - 3 = \frac{4}{3}x - \frac{8}{3} \rightarrow y = \frac{4}{3}x + \frac{1}{3}$

4) (4 points) Remanufacturing diesel engines has a cost function $C(e) = 7e + 5600$ and a revenue function $R(e) = 10e$ where $e$ is the number of engines and everything is in units of US dollars.
(a) What is the amount of the Fixed Costs?
(b) What is the value of the Marginal Cost at $e$ (i.e. difference between costs at $e+1$ and $e$)?
(c) What is the value of the Marginal Revenue at $e$?
(d) Give the Profit Function $P(e)$ (in simplified form please).
(e) How many engines need to be remanufactured to break even?

(a) Fixed cost $= C(e=0) = 7 \cdot 0 + 5600 = 5600$

(b) marginal cost at $e = \frac{C(e+1) - C(e)}{(e+1) - e} = \frac{(7(e+1)+5600) - (7e+5600)}{1}$

$= 7$ ($7 per unit)

(c) similar... marginal revenue $= \frac{R(e+1) - R(e)}{(e+1) - e} = \ldots = 10$

(d) $P(e) = R(e) - C(e) = 10e - (7e + 5600) \Rightarrow$

$P(e) = 3e - 5600$

(e) break even $\Rightarrow$ Profit $P(e) = 0$ so $3e - 5600 = 0$

so $e = \frac{5600}{3} = 1866.66\ldots$ (we need to make 1867 engines to break even)

5) (5 points) A local brewer wants to create two ales for Spring semester: the "Freshman15" and "Spring Delight". The profit is \$7 per case of SDelight versus \$5 per case of Freshman15.

The demand for SDelight is at least twice as much as the demand for Freshman15. Realizing this, the manufacturer decides to make SDelight a limited edition to preserve its special character. The *maximum* number of cases of SDelight per month is 500 plus an extra allowance of 1 case of SDelight for every 4 cases of Freshman15 produced (he can always produce less SDelight than this, perhaps even zero SDelight).

The manufacturer can produce at most 1000 total cases per month. What is the maximum profit and how many cases of each brew are produced?

let $x$ = # cases Delight, $y$ = # cases of Freshman

Total profit = $7x + 5y = P(x,y)$

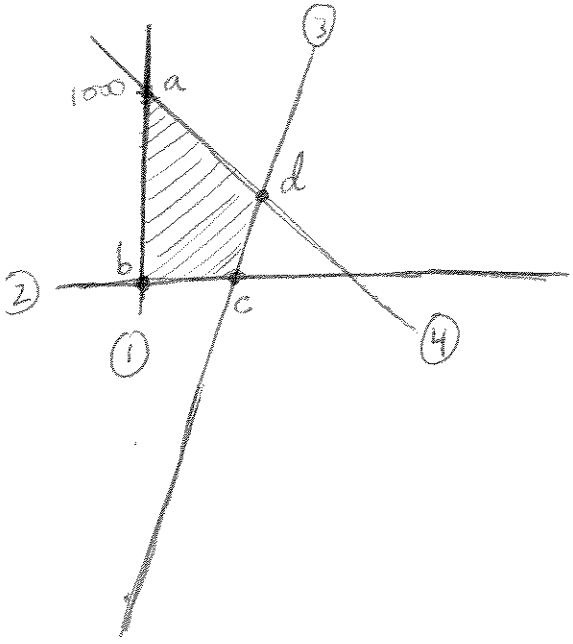
constraints

1) $x \geq 0$

2) $y \geq 0$

3) $x \leq 500 + \frac{1}{4}y \longrightarrow y \geq 4x - 2000$

4) $x + y \leq 1000 \longrightarrow y \leq -x + 1000$




points $a = (0, 1000)$ (set $x=0$ in eqn 4)

$b = (0,0)$

$c = (500, 0)$ (set $y=0$ in eqn 3)

$d$: 3 = 4 ← (solve to find point)

$4x - 2000 = -x + 1000$

$5x = 3000 \longrightarrow x = 600$

$(x=600, y=400)$

Maximum value of $P(x,y)$ is on one of these points, plug in and check

(a) $P(0, 1000) = 5000$

(b) $P(0,0) = 0$

(c) $P(500, 0) = 3500$

(d) $P(600, 400) = 6200$

∴ max profit is \$6200 at 600 Delight, 400 Freshman

6) (2 points) Consider the following matrices

$$A = \begin{pmatrix} 3 & 7 & 1 & 2 \\ 10 & 1 & 0 & 2 \\ 5 & 8 & 1 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 0 & 1 \end{pmatrix} \tag{1}$$

Compute $A + B^T$

$$\begin{pmatrix} 3 & 7 & 1 & 2 \\ 10 & 1 & 0 & 2 \\ 5 & 8 & 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 0 \\ 3 & 6 & 9 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 11 & 8 & 12 \\ 12 & 6 & 8 & 2 \\ 8 & 14 & 10 & 4 \end{pmatrix}$$

7) (4 points) Using the matrices in problem 6 compute $A \cdot B$

$$\begin{pmatrix} 3 & 7 & 1 & 2 \\ 10 & 1 & 0 & 2 \\ 5 & 8 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 0 & 1 \end{pmatrix} =$$

$3 \times 4 \quad 4 \times 3$

$$= \begin{pmatrix} 3\cdot1+7\cdot4+1\cdot7+2\cdot10 & 3\cdot2+7\cdot5+1\cdot8+2\cdot0 & 3\cdot3+7\cdot6+1\cdot9+2\cdot \\ 10\cdot1+1\cdot4+0\cdot7+2\cdot10 & 10\cdot2+1\cdot5+0\cdot8+2\cdot0 & 10\cdot3+1\cdot6+0\cdot9+2\cdot1 \\ 5\cdot1+8\cdot4+1\cdot7+3\cdot10 & 5\cdot2+8\cdot5+1\cdot8+3\cdot0 & 5\cdot3+8\cdot6+1\cdot9+3\cdot1 \end{pmatrix}$$

$$= \begin{pmatrix} 58 & 49 & 62 \\ 34 & 25 & 38 \\ 74 & 58 & 75 \end{pmatrix}$$