1.2

When multiplying inequalities by a negative, the direction of the inequality must change ($-x \geq -1$ --> $x \leq 1$)

1.3

EQUATION OF A LINE

Point-Slope Form: Given a point (x_1, y_1) and the slope, m, of the line, the line equation is $y - y_1 = m(x - x_1)$.

Slope-Intercept Form: Given the slope, m, of the line and the y-intercept, $(0, b)$, the line equation is $y = mx + b$.

PARALLEL LINES

Two lines are parallel if and only if the slopes of the lines are the same, i.e., $m_1 = m_2$.

PERPENDICULAR LINES

Two lines are perpendicular if and only if the slopes of the lines are the negative reciprocals of one another, i.e., $m_1 = \frac{-1}{m_2}$ and $m_2 = \frac{-1}{m_1}$. (*Note:* Horizontal and vertical lines are also perpendicular to each other, even though they do not fit this definition because vertical line slopes are undefined.)

SLOPE OF A LINE

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Note: For a horizontal line, the slope is 0. For a vertical line, the slope is undefined.

1.5

Profit = revenue - cost
Marginal revenue = slope of revenue line/function
Marginal profit = slope of profit line/function

Quadratic Equation:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2.1

a) a^{-1} in a 2x2 matrix = $\frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

b) Matrix MUST be square to have an inverse

3.2

Domain restrictions: x-value may not create 0 denominator or a negative $\sqrt{\ }$

To be a function, x must only have one solution, i.e. if substituting "1": $y^2 = 3x^2$ --> $y^2 = 3(1^2)$ --> $y^2 = \sqrt{3}$ ---> $y = \pm 3$

3.3

- Vertex = -b/2a
- Axis of Symmetry = vertical line that goes through vertex or x coordinate
- Parabola is concave down if first / highest coefficient is negative
  x intercepts = roots or zeros: substitute y = 0

3.4

- Break-even: C(x) = R(x), or cost = revenue
- Profit function: P(x) = R(x) – C(x), or revenue minus profit
- Maximum Units: find vertex of P(x)
- Maximum Profit: substitute x-value of vertex back into the profit function

3.5

- x - intercept: find by setting y = 0
- y - intercept: find by setting x = 0

	POSITIVE LEADING COEFFICIENT	NEGATIVE LEADING COEFFICIENT
Odd degree polynomial	Goes up on right and down on left side of graah	Goes down on right and up on left side of graph
Even degree polynomial	Goes up on both sides of graph	Goes down on both sides of graph

3.6

To find the horizontal asymptotes of a rational function:

1. **If the degree of $n(x)$ > degree of $d(x)$, then there is no horizontal asymptote.**
2. **If the degree of $n(x)$ < degree of $d(x)$, then the horizontal asymptote is the line $y = 0$.**
3. **If the degree of $n(x)$ = degree of $d(x)$, then the horizontal asymptote is the line $y = \frac{a}{b}$, where a = leading coefficient of $n(x)$ and b = leading coefficient of $d(x)$.**

4.1

Finding an inverse function:

"Pants" Technique: place the function in reverse order, using the inverse of each function (i.e. ?x = x2 , -2 = 2, etc.)

Algebraic Technique:

a) First, exchange the x and y variables in the function
b) Use algebraic operations to get *y* by itself on one side, and *x* on the other.

4.3

- $f(x) = a^x$ and $f^{-1}(x) = \log_a x$
- $f(f^{-1}(x)) = f(\log_a x) = a^{\log_a x} = x$ and $f^{-1}(a^x) = \log_a a^x = x$

4.5

CHANGE OF BASE FORMULA

For $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$, we can change logarithm according to

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}.$$

5.2

- Simple Interest: $S = P(1+rt)$
- Compound Interest: $S = P(1 + r/n)^{nt}$
- Continuous Compounding: $S = Pe^{rt}$
- Annual Percentage Yield: $APY = (1 + r/n)^{n} - 1$ (periodic compounding)
- Annual Percentage Yield: $APY = e^{r} - 1$ (continuous compounding)

5.3

PRESENT VALUE OF AN ORDINARY ANNUITY

Let R = the withdrawal/payment amount, n = the number of compoundings per year, r = the annual interest rate, t = total number of years for withdrawals/payments, N = the total number of withdrawals/payments = nt, $r_c = \frac{r}{n}$ = the interest rate per compounding period. Then the present value S of the account is $S = \frac{R(1-(1+r_c)^{-N})}{r_c}$. (*Note:* The number of compoundings per year must match the number of payments/withdrawals made per year.)

PRESENT VALUE OF AN ANNUITY DUE

Let R = the withdrawal/payment amount, n = the number of compoundings per year, r = the annual interest rate, t = total number of years for withdrawals/payments, N = the total number of withdrawals/payments = nt, $r_c = \frac{r}{n}$ = the interest rate per compounding period. Then the present value S_{due} of the account is $S_{due} = \frac{R(1+r_c)(1-(1+r_c)^{-N})}{r_c} = (1+r_c)S$, where S is the present value of the corresponding ordinary annuity.

PRESENT VALUE OF A DEFERRED ANNUITY

Let P = present value of a deferred annuity, where it's deferred (with no withdrawals made) for m compounding periods, at which point N regular withdrawals of \$$R$ take place.

$$P = \frac{R(1-(1+r_c)^{-N})}{r_c(1+r_c)^m}$$

SINKING FUND PAYMENT FORMULA

The formula gives the regular payment, R, necessary to invest in an account to reach S dollars at the end of N compounding periods.

$$R = S\left(\frac{r_c}{(1+r_c)^N - 1}\right)$$

5.4

FUTURE VALUE OF AN ORDINARY ANNUITY

Let N be the total number of payments made into the annuity and R be the regular payment amount. Then the future value of an ordinary annuity is

$$S = R\left(\frac{\left(1+\frac{r}{n}\right)^N - 1}{\frac{r}{n}}\right) = \frac{Rn}{r}\left(\left(1+\frac{r}{n}\right)^N - 1\right).$$

Another way to write this same formula is to define a new variable $r_c = \frac{r}{n}$, which is the interest rate for each compounding period. That transforms our formula to $S = R\left(\frac{(1+r_c)^N - 1}{r_c}\right)$. (*Note:* The number of payments made per year must match with the number of compoundings per year in order to use this formula!)

FUTURE VALUE OF AN ANNUITY DUE

Let N be the total number of payments made into the annuity, R is the payment amount and $r_c = \frac{r}{n}$ is the interest rate per compounding period. Then S_{due} is the future value of the annuity given by $S_{due} = R(1+r_c)\left(\frac{(1+r_c)^N - 1}{r_c}\right) = S(1+r_c)$ (where S = the future value of the corresponding ordinary annuity).

5.5

AMORTIZATION FORMULAS

Let r = annual interest rate, n = the number of compoundings per year, $r_c = \frac{r}{n}$ = the interest rate per payment, t = number of years for payments, N = total number of payments = nt, k = number of payments made so far, S = loan amount and R = payment amount.

(a) Periodic Payment of Amortized Loan: $R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)$

(b) Total Interest Paid: $NR - S$ (the total money paid minus the loan amount)

(c) Loan Payoff Amount: Payoff = $S_{N-k} = R\left(\frac{1-(1+r_c)^{-(N-k)}}{r_c}\right)$

If k payments have occurred so far and a total of N payments are due, then there are $(N-k)$ payments missing from the loan. (This is simply the formula for the present value of an ordinary annuity needed to produce those $(N-k)$ payments that will be paid as a lump sum.)