M1060-2 QUIZ 5 (Spencer Stirling) - October 7, 2010

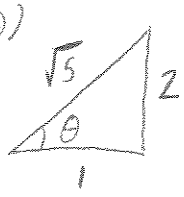
Directions: You may attach more sheets if necessary. SHOW ALL WORK and CLEARLY mark your solutions.

1) (6 points) Find the exact value of each expression (hint: NO calculator. sketch a right triangle)

(a) $\cos(\arctan(2))$

Let $\theta = \arctan(2)$, so $\tan(\theta) = 2$ and θ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Want $\cos(\theta)$

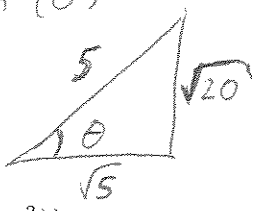


so $\cos(\theta) = \frac{1}{\sqrt{5}}$

(b) $\sin(\arccos(\frac{\sqrt{5}}{5}))$

Let $\theta = \arccos\left(\frac{\sqrt{5}}{5}\right)$, so $\cos\theta = \frac{\sqrt{5}}{5}$ and θ in $[0, \pi]$

Want $\sin(\theta)$

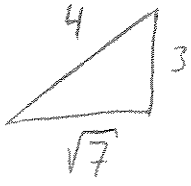


so $\sin(\theta) = \frac{\sqrt{20}}{5}$

(c) $\tan(\arcsin(-\frac{3}{4}))$

Let $\theta = \arcsin\left(-\frac{3}{4}\right)$, so $\sin(\theta) = -\frac{3}{4}$ and θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Want $\tan(\theta)$



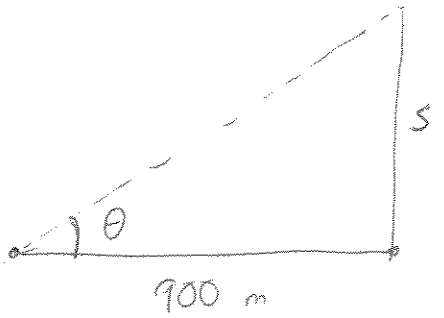
so $\tan(\theta) = \pm \frac{3}{\sqrt{7}}$?

Now since $\sin(\theta) = -\frac{3}{4}$ and θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ we have θ in $\left[-\frac{\pi}{2}, 0\right]$. Hence $\tan(\theta)$ is negative

so $\tan(\theta) = -\frac{3}{\sqrt{7}}$

2) A television camera at ground level is filming the lift-off of a space shuttle. The camera is 900 meters away from the launch pad. Let θ be the angle of elevation of the shuttle, and let s be the height of the shuttle. Assume that the shuttle is accelerating at 50 m/s^2.

(a) (2 points) Write θ as a function of s



we have $\tan\theta = \frac{s}{900\ m}$

$\Rightarrow \theta = \arctan\left(\frac{s}{900\ m}\right)$

(b) (2 points) Find θ when $s = 400$ meters and also when $s = 1500$ meters

$s = 400\ m$

$\theta = \arctan\left(\frac{400\ m}{900\ m}\right) \approx 23.96^\circ$ or 0.4181 radians

$s = 1500\ m$

$\theta = \arctan\left(\frac{1500\ m}{900\ m}\right) \approx 59.04^\circ$ or 1.03 radians